and $(\partial H/\partial T_s)_P$ values. Measurement of the shift in transition temperatures caused by external magnetic fields of different orientations to the tetragonal c axis has provided information on crystalline anisotropy of the intermediate magnetic state in agreement with the results from magnetization measurements.

Measurements of the Hall effect have shown that the number of current carriers is the same in both the ferrimagnetic and antiferromagnetic states, and this information has yielded a value of 2.5 eV for the Fermi energy at absolute zero.

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Effect of Dipolar Fields on the Antiferromagnetic Spin-Wave Spectrum

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The spin-wave dispersion relation for a simple uniaxial antiferromagnet is calculated, with inclusion of volume dipolar fields. The spin-wave spectrum is anisotropic and a degeneracy exists between the uniform antiferromagnetic-resonance modes and manifolds of short-wavelength spin waves, similar to the situation in a ferromagnet. The densities of states of these degenerate spin waves are calculated, and are then used to estimate antiferromagnetic-resonance linewidths caused by paramagnetic impurities having a different g value or exchange coupling to the host spins. Finally, the linewidth caused by surface pits in the sample is estimated.

I. INTRODUCTION

THIS paper is concerned with the effects of the classical dipole-dipole interaction between magnetic moments on the spin-wave spectrum of a simple uniaxial antiferromagnet. In ferromagnets, it has been known for some time[1] that the surface demagnetizing fields have a profound influence on the ferromagnetic-resonance frequency. Further, it has been shown by Herring and Kittel[2] that the volume dipolar fields cause the ferromagnetic spin-wave spectrum to become anisotropic. Anderson and Suhl[3] were the first to recognize that the surface demagnetizing fields give rise to a region of the spin-wave spectrum which is degenerate in frequency with the uniform ferromagnetic-resonance mode. The reason for this degeneracy is that, for spin waves with wavelengths short compared to sample dimensions ($kL \gg 1$), the signs of the magnetic poles on the surface giving rise to the transverse demagnetizing field oscillate rapidly in space. The transverse demagnetizing field is proportional to the number of uncompensated poles and is, therefore, small. On the other hand, the longitudinal demagnetizing field is large and leads to a reduction of the spin-wave frequencies so that some of the spin waves become degenerate with the uniform mode. This degeneracy allows crystalline imperfections[4] to mix the uniform mode ($k=0$) spin waves generated on resonance with the plane-wave ($k \neq 0$) spin waves in the degenerate manifold and thus gives rise to an important source of linewidth. Further, it has been shown by Suhl[5] that the premature saturation of the ferromagnetic-resonance[6] signal is due to coherent spin-wave scattering into the degenerate modes via the dipolar-anisotropy fields.

It is interesting to speculate about the importance of similar effects in antiferromagnets where there is no net magnetization and thus no bulk demagnetizing field. Keffer and Kittel[7] have indeed shown that the oscillating transverse moment generated in antiferromagnetic resonance (AFR) does give rise to a demagnetization shift in the antiferromagnetic-resonance frequency. This shift is, however, quite small compared to the ferromagnetic case. In ferromagnets $\delta\omega/\omega \approx NM_s/H$, which may be of the order of unity, where N is some appropriate demagnetization factor, M_s the macroscopic mag-

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netization and H the sum of the external field and internal-anisotropy field. The corresponding result in antiferromagnets is $\delta\omega/\omega \approx N_\perp M_s/H_e$, where now M_s is the sublattice magnetization and H_e is the effective exchange field. For $N_\perp M_s \approx 10^3$ Oe, $He \approx 10^6$ Oe, $\delta\omega/\omega \approx 3\times 10^{-2}$, which is about one hundred times smaller than the ferromagnetic resonance results. In Sec. II, we show that, nevertheless, in antiferromagnets the dipolar fields do produce an anisotropic spin-wave spectrum with a degenerate manifold. Section III deals with some order of magnitude calculations for AFR linewidths arising from scattering from crystalline imperfections. It is shown that, in contrast to the ferromagnetic situation, paramegnetic impurities with a different exchange coupling to the host spins may give rise to scattering from the uniform mode.

With the recent discovery of several antiferromagnets with resonance frequencies in the microwave range (e.g., $RbMnF_3$, $KMnF_3$, etc.) it is hoped that careful measurements may allow the observation of such dipolar effects. Indeed, there is already evidence[8] of premature saturation of the AFR signal.

II. SPIN-WAVE SPECTRUM

In this section, we treat the problem of determining the frequency of the kth Fourier component of a small fluctuation in the transverse magnetization of an antiferromagnet. The dipolar fields are introduced by requiring that Maxwell's equations be satisfied in the limit of vanishingly small electric and displacement currents.[2] Ziman[9] has formally solved the problem of the dipolar effects on the ($k\neq 0$) part of the spin-wave spectrum in terms of a canonical transformation on the spin-wave creation and annihilation operators. In the Appendix, we give an entirely equivalent treatment by solving for the eigenfrequencies of the coupled spin-wave equations of motion in the presence of a classical dipole-dipole interaction.

If $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetizations of the two sublattices, we may write their equations of motion in the absence of dipolar interactions as

$$\begin{aligned} d\mathbf{M}_1/dt &= \gamma \mathbf{M}_1 \times \mathbf{H}_{\text{eff}}(1), \\ d\mathbf{M}_2/dt &= \gamma \mathbf{M}_2 \times \mathbf{H}_{\text{eff}}(2), \end{aligned} \tag{II.1}$$

where γ is the gyromagnetic ratio and $\mathbf{H}_{\text{eff}}(i)$ is the effective field at the ith sublattice, including the external field $\mathbf{H}_0$ directed along the easy (z) axis, an internal single-ion anisotropy field, $\mathbf{H}_A$, arising, for example, from the combined crystalline field and spin orbit interaction, and the exchange field experienced by a spin on one sublattice arising from its nearest neighbor spins on the other sublattice. Then the equations of motion become

$$\begin{aligned} d\mathbf{M}_1/dt &= \gamma \mathbf{M}_1 \\ &\quad \times [\mathbf{H}_0 + \mathbf{H}_A - (H_e/M_s)(1+b^2\nabla^2)\mathbf{M}_2], \\ d\mathbf{M}_2/dt &= \gamma \mathbf{M}_2 \\ &\quad \times [\mathbf{H}_0 - \mathbf{H}_A - (H_e/M_s)(1+b^2\nabla^2)\mathbf{M}_1], \end{aligned} \tag{II.2}$$

where $b = az^{-1/2}$; a is the nearest neighbor separation and z is the number of nearest neighbors. The linearized equations for small transverse fluctuations in the magnetization become

$$\begin{aligned} &(1/\gamma) dM_1^+/dt \\ &\quad = -i[M_1^+(H_0 + H_A + H_e) + H_e(1+b^2\nabla^2)M_2^+], \\ &(1/\gamma) dM_2^+/dt \\ &\quad = -i[M_2^+(H_0 - H_A - H_e) - H_e(1+b^2\nabla^2)M_1^+], \end{aligned} \tag{II.3}$$

where $M^+ = M_x + iM_y$. If we take M^+ to vary as $e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$ and solve the resulting secular determinant, we obtain the well-known[10] AF dispersion law:

$$\omega/\gamma = H_0 \pm (H_A^2 + 2H_eH_A + 2H_e^2b^2k^2)^{1/2}. \tag{II.4}$$

In the absence of an external field, there exist two degenerate, oppositely rotating modes for a given wave vector. The degeneracy is removed by the external field.

We shall now include the effects of the dipolar fields. For the uniform ($k=0$) modes, there is a surface demagnetizing field $-\mathbf{NM}$, where $\mathbf{N}$ is the demagnetization tensor, and $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$. For simplicity, we assume a spheroidal sample with the easy axis colinear with the axis of revolution. Then $\mathbf{N}$ is diagonal, with $N_x = N_y = N_\perp$. For the other modes, (i.e., $kL \gg 1$) the transverse component of the magnetization varies sufficiently rapidly through the sample for the transverse demagnetizing field to average to zero. The only contribution is then $-N_zM_z$ along the z axis. However, to first order in small transverse fluctuations of the magnetization $M_z = M_{1z} - M_{2z}$ which vanishes. Consequently, in the $k\neq 0$ part of the spin-wave spectrum, there will be no surface demagnetization corrections. We now investigate the volume dipolar interactions. Following Kittel and Herring,[2] neglecting any currents, we have the Maxwell equations

$$\begin{aligned} \nabla\cdot\mathbf{B} &= 0, \\ \nabla\times\mathbf{H}_s &= 0, \end{aligned} \tag{II.5}$$

where $\mathbf{B} = \mathbf{H}_s + 4\pi\mathbf{M}$ and $\mathbf{H}_s$ is the dipolar field caused by the spins themselves. If we try plane-wave solutions of the form

$$\mathbf{M}_\perp = \mathbf{M}_\perp(0) e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}, \tag{II.6}$$

where $\mathbf{M}_\perp = \hat{\imath}(M_1^x + M_2^x) + \hat{\jmath}(M_1^y + M_2^y)$, (5) gives the dipolar field as

$$\mathbf{H}_s = -(4\pi/k^2)(\mathbf{k}\cdot\mathbf{M}_\perp)\mathbf{k}. \tag{II.7}$$

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Now, for the uniform mode, including the surface demagnitizing fields, the equations of motion become

$$(\omega/\gamma)M_1^+ = M_1^+(H_0+H_A+H_e+N_\perp M_s) + M_2^+(H_e+N_\perp M_s), \quad \text{(II.8)}$$

$$(\omega/\gamma)M_2^+ = M_2^+(H_0-H_A-H_e-N_\perp M_s) - M_1^+(H_e+N_\perp M_s).$$

The eigenfrequencies of this secular equation have been given by Keffer and Kittel[7] and are

$$\omega/\gamma = H_0 \pm [H_A^2+2H_A(H_e+N_\perp M_s)]^{1/2}. \quad \text{(II.9)}$$

This is essentially the result given in the Introduction. For the sufficiently short-wavelength spin waves, the equations of motion, including the volume dipolar fields, are

$$d\mathbf{M}_1/dt = \gamma\mathbf{M}_1\times[\mathbf{H}_0+\mathbf{H}_A-(H_e/M_s)(1+b^2\nabla^2)\mathbf{M}_2 - (4\pi/k^2)(\mathbf{k}\cdot\mathbf{M}_\perp)\mathbf{k}], \quad \text{(II.10)}$$

$$d\mathbf{M}_2/dt = \gamma\mathbf{M}_2\times[\mathbf{H}_0-\mathbf{H}_A-(H_e/M_s)(1+b^2\nabla^2)\mathbf{M}_1 - (4\pi/k^2)(\mathbf{k}\cdot\mathbf{M}_\perp)\mathbf{k}].$$

The linearized equation of motion resulting from (10) cannot be expressed simply in terms of M^+. The dipolar terms cause mixing of M^+ and M^-, i.e., the normal modes become elliptically polarized. The eigenfrequency solutions of this 4×4 secular equation are

$$(\omega/\gamma)^2 = H_0^2+H_A^2+2H_eH_A+2H_e^2b^2k^2 + 4\pi M_s\sin^2\theta_k(H_A+H_eb^2k^2) \pm\{4H_0^2[2H_e^2b^2k^2+H_A^2+2H_eH_A + 4\pi M_s\sin^2\theta_k(H_A+H_eb^2k^2)] + 16\pi^2M_s^2H_A\sin^4\theta_k(H_A+2H_eb^2k^2)\}^{1/2}, \quad \text{(II.11)}$$

where θ_k is the angle between the z axis and the direction of propagation of the spin wave $\mathbf{k}$. In the absence of an external field, the complicated result (11) can be considerably simplified to give the two frequencies:

$$\omega_1/\gamma = (H_A^2+2H_eH_A+2H_e^2b^2k^2)^{1/2},$$

$$\omega_2/\gamma = [H_A^2+2(H_A+H_eb^2k^2)\times(H_e+4\pi M_s\sin^2\theta_k)]^{1/2}. \quad \text{(II.12)}$$

Notice the interesting result that one normal frequency is completely independent of any dipolar interactions. This result can be understood in the following manner. In the absence of external and dipolar fields, there exist two degenerate oppositely rotating modes. In the presence of the dipolar interaction, these two modes (for a given $\mathbf{k}$) can mix in such a way that $\mathbf{M}_\perp$ is always perpendicular to $\mathbf{k}$ and thus the dipolar field vanishes. All the dipolar energy is then taken up by the other mode. In Fig. 1, we give schematically, the zero-field spin-wave spectrum. Notice that, except for the pathological situation of an infinitely thin disk, there exist spin waves degenerate with the uniform mode. An external field, H_0, splits the degeneracy of the two oppositely polarized modes for a given wave vector and then the two modes will share the dipolar energy. However, for the special case of longitudinal spin waves, $\theta_k=0$, the spectrum is just that given by (4) and there are no dipolar effects. In general, the transverse magnons, $\theta_k=\frac{1}{2}\pi$, have the largest dipolar energies because for these magnons $\mathbf{k}\cdot\mathbf{M}_\perp$ is maximum. In Fig. 2 the spin-wave spectrum is shown with a nonvanishing external field [assuming that it is smaller than the "flopping" field,[10] $(2H_eH_A)^{1/2}$].

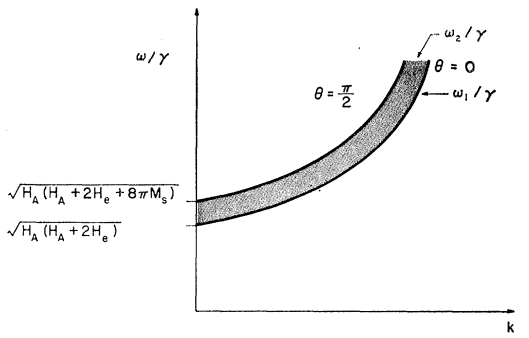

FIG. 1. Schematic representation of the antiferromagnetic spin-wave spectrum in zero external field.

III. DENSITY OF DEGENERATE STATES AND LINEWIDTH ARISING FROM IMPERFECTIONS

Several authors[4,11–13] have studied the mixing by crystal imperfections of the uniform ferromagnetic resonance mode with the degenerate (or S) modes. The importance of aperiodic imperfections is that they give rise to spin-wave scattering processes which do not conserve wave vector. Such scattering from the uniform mode into the degenerate manifold has been recognized as playing a central role in ferromagnetic relaxation in insulators. Sparks, Loudon, and Kittel[13] have shown that the low-temperature linewidth in ultra-pure YIG may be understood in terms of such mixing via the dipolar fields associated with surface pits. Callen and his co-workers[11,12] have discovered several mechanisms for the mixing of the uniform mode with S magnons through point imperfections, concentrating mainly on the effects of spatial fluctuations of the spin-orbit coupling in the disordered ferrites. In this section, we shall derive the expressions for the relevant densities of degenerate states for scattering of spin waves out of the uniform mode by imperfections in antiferromagnets. We shall then use these results to estimate the AFR linewidth arising from some types of point imperfections and pits.

The scattering process about an imperfection may be thought of as a process in which a uniform-mode magnon is destroyed and a degenerate magnon is created. The

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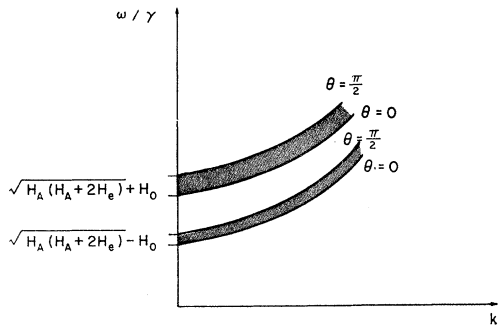

FIG. 2. Antiferromagnetic spin-wave spectrum for finite external field.

scattering potential is written as

$$V(\mathbf{k})=F(\mathbf{k})(\alpha_0\alpha_\mathbf{k}^{+}+\alpha_\mathbf{k}\alpha_0^{+}), \qquad \text{(III.1)}$$

where $F(\mathbf{k})$ is the strength of the interaction, depending on the type of imperfection. The α_0 and α_0^{+} are destruction and creation operators for uniform mode magnons; they obey boson commutation relations. The $\alpha_\mathbf{k}$'s are similar creation and destruction operators for the appropriate degenerate excitations. The net rate at which uniform-mode magnons are scattered into a degenerate state of wave vector $\mathbf{k}$ is given by

$$W=(2\pi/\hbar)[|\langle n_\mathbf{k}+1, n_0-1|V(\mathbf{k})|n_\mathbf{k}, n_0\rangle|^2 - |\langle n_\mathbf{k}-1, n_0+1|V(\mathbf{k})|n_\mathbf{k}, n_0\rangle|^2]\rho_\mathbf{k}, \qquad \text{(III.2)}$$

or, using (III.1),

$$W=(2\pi/\hbar)|F(\mathbf{k})|^2(n_0-n_\mathbf{k})\rho_\mathbf{k}, \qquad \text{(III.3)}$$

where $n_\mathbf{k}$ is the occupation number of the state $\mathbf{k}$, and $\rho_\mathbf{k}$ is the number of states per unit energy range with wave vector $\mathbf{k}$. The thermal equilibrium occupation of the degenerate states is the same as for the uniform mode. If the degenerate modes are assumed to be in thermal equilibrium with the lattice, then

$$W=(2\pi/\hbar)|F(\mathbf{k})|^2\rho_\mathbf{k}(n_0-\bar{n}_0), \qquad \text{(III.4)}$$

where $\bar{n}_0$ is the thermal-equilibrium value of n_0. The relaxation time for scattering out of the uniform mode into the degenerate states is then

$$\tau^{-1}=(V/4\pi^2\hbar)\int|F(\mathbf{k})|^2\rho_\mathbf{k}d\mathbf{k}, \qquad \text{(III.5)}$$

where V is the sample volume. The linewidth is related to the relaxation time by $\Delta H=(\gamma\tau)^{-1}$.

The evaluation of the density of states for the spectrum (II.11) is fairly complicated. However, for point imperfections, the strength of the scattering potential, $F(\mathbf{k})$, is very nearly independent of $\mathbf{k}$. Then it is not a bad approximation[4] to neglect the anisotropic nature of the spectrum and to assume that all the degenerate S magnons lie at the upper limit of the degenerate spectrum in k space, i.e., at k_m, given by

$$2H_e^2b^2k_m^2=2N_\perp H_AM_s, \qquad \text{(III.6)}$$

where the external field is taken to be zero. The density of states is

$$V(2\pi)^{-3}\rho_\mathbf{k}d\mathbf{k}=V(2\pi)^{-3}4\pi k^2\delta(\hbar\omega-\hbar\omega_0)dk. \qquad \text{(III.7)}$$

Using (II.11) and (III.6) one finds

$$V(2\pi)^{-3}\rho_\mathbf{k}d\mathbf{k}=(V/2\pi^2)(k_m/2\hbar\gamma H_e^2b^2)\times(2H_eH_A)^{1/2}\delta(k-k_m)dk. \qquad \text{(III.8)}$$

This is smaller than the corresponding ferromagnetic density of states by approximately $H_A/H_e\approx10^{-6}$–10^{-2}. This might appear to indicate that the linewidths arising from impurity scattering are smaller in an antiferromagnet than in a ferromagnet. This is, however, not the case and in fact the amplitude of the spin motion for a single antiferromagnetic excitation exceeds that of a ferromagnet by just the appropriate factor to give the same order of magnitude for the linewidth for similar processes.

For extended imperfections, such as surface pits, the potential strength $F(\mathbf{k})$ may be a very sensitive function of kR, where R is the characteristic dimension of the imperfection. In the absence of an external field, (III.8) remains the correct density of states for the lower branch of the spectrum. For the upper branch, specializing now to the case of a spherical specimen where $N=\frac{4}{3}\pi$, the density of states is

$$V(2\pi)^{-3}\rho_\mathbf{k}d\mathbf{k}=\frac{V(2H_eH_A)^{1/2}k^2\delta(\cos\theta_k-\cos\phi)d(\cos\theta_k)dk}{4\pi^2\gamma\hbar\{(2H_eH_A+2H_e^2b^2k^2)(4\pi M_s/H_e)[2H_e^2b^2k^2+(16\pi H_AM_s/3)]\}^{1/2}}, \qquad \text{(III.9)}$$

where

$$\cos^2\phi\approx[2H_e^2b^2k^2+(16\pi H_AM_s/3)]/(4\pi M_s/H_e)(2H_eH_A+2H_e^2b^2k^2). \qquad \text{(III.10)}$$

In order to write the scattering potential in the form (III.1) we introduce the antiferromagnetic spin-wave creation and annihilation operators for the two branches of the spectrum, $\alpha_\mathbf{k}$, $\alpha_\mathbf{k}^{\dagger}$, $\beta_\mathbf{k}$, and $\beta_\mathbf{k}^{\dagger}$. These operators obey Bose commutation rules and are defined by

$$\alpha_\mathbf{k}=u_k\alpha_\mathbf{k}+v_k\beta_\mathbf{k}^{\dagger}; \quad b_\mathbf{k}=v_k\alpha_\mathbf{k}^{\dagger}+u_k\beta_\mathbf{k}, \qquad \text{(III.11)}$$

where

$$S_i^z=S-a_i^{\dagger}a_i, \quad S_j^z=-S+b_j^{\dagger}b_j \qquad \text{(III.12)}$$

for the two sublattices. The $a_\mathbf{k}$'s and $b_\mathbf{k}$'s are Fourier transforms of the a_i's and b_i's, and the canonical transformation coefficients (in the absence of dipolar fields) u_k and v_k are given by

$$u_k^2=\tfrac{1}{2}\{1+\gamma(H_e+H_A)/|\omega(k)|\}, \quad v_k^2=\tfrac{1}{2}\{-1+\gamma(H_e+H_A)/|\omega(k)|\}, \qquad \text{(III.13)}$$

where $\omega(k)$ is given by (II.4) for zero external field.

As a typical example of scattering from a point imperfection with a wave-vector-independent perturbation, we consider the case of a paramagnetic impurity whose g value is different from that of the host spins. Then the scattering potential is

$$\mathcal{H}'=\beta H(gS_i^z-g'S'^z_i), \qquad \text{(III.14)}$$

where the impurity spin has $g'\neq g$. Then, for $S'=S$, and using (III.11), (III.12), and (III.13) together with the density of states (III.8), we obtain

$$\tau^{-1}=(16\pi\hbar^2\gamma H_e)^{-1}(g'-g)^2\beta^2H^2 \times(2N_\perp M_s/H_e)^{1/2}fz^{3/2}, \qquad \text{(III.15)}$$

where f is the fractional number of imperfections and z is the number of nearest neighbors. Notice that the result is independent of the anisotropy field as was previously indicated. For $H_e\approx10^6$ Oe, $M_s\approx500$ Oe, $H\approx10^4$ Oe, $g'-g\approx1$, $z=8$, (III.15) gives a linewidth of the order of 10^{-2} Oe per percent impurity. This result (III.15) is essentially the same as for the corresponding ferromagnetic scattering.

Another interesting point imperfection scattering mechanism arises from a paramagnetic impurity which is coupled to the host spins by a different exchange integral J. Such an imperfection is ineffective in scattering uniform mode spin waves in a ferromagnet because an isotropic exchange interaction commutes with the total spin S [$\mathbf{S}^2=(\sum_i\mathbf{S}_i)^2$] of the sample which is a good quantum number. The total spin of the sample decreases by one unit when a uniform mode magnon is destroyed and thus exchange impurities cannot relax the uniform mode. This argument is no longer valid for an antiferromagnet because the crystalline field anisotropy does not commute with S; thus the anisotropy allows an exchange imperfection to relax the uniform mode. For such a process the scattering potential may be written

$$\mathcal{H}'=2J\sum_\delta(\mathbf{S}_i\cdot\mathbf{S}_\delta)-2J'\sum_\delta(\mathbf{S}_i'\cdot\mathbf{S}_\delta), \qquad \text{(III.16)}$$

where the sum over δ represents a sum over the nearest neighbors of S_i', the impurity spin. For $S'=S$, this gives a relaxation rate

$$\tau^{-1}=(z^2S^2H_A^2/\pi\hbar^2\gamma H_e^3)(J'-J)^2 \times(2N_\perp M_s/H_e)^{1/2}fz^{3/2}, \qquad \text{(III.17)}$$

where z is the number of nearest neighbor spins for a given spin. Notice that the linewidth arising from this process vanishes as the anisotropy field tends toward zero. For a typical antiferromagnet such as MnF_2, where $H_A\approx10^4$ Oe, $He\approx10^6$ Oe, $M_s\approx500$ Oe, $z=8$ and $J'-J\approx10^{-14}$ erg, this gives a linewidth of the order of 10 Oe per percent impurity.

Sparks, Loudon, and Kittel[13] have calculated the linewidth arising from spin-wave scattering via the dipolar fields associated with surface pits in ferrimagnetic yttrium iron garnet. For a spherical YIG sample, assumed completely convered with hemispherical surface pits, their result for the linewidth is $\Delta H=\frac{1}{2}M_s(R/r_0)$, where R is the mean pit radius and r_0 the sample radius. For $R\approx10^{-4}$ cm, $r_0\approx10^{-2}$ cm, this gives a linewidth of several Oe. For an antiferromagnet, the same mechanism may occur with some differences in detail. For example, the scattering may mix the two spin-wave branches. Also, there is only a transverse moment (generated at resonance) to which the dipolar fields associated with the pit may couple. This moment[7] is of the order of $(H_A/H_e)^{1/2}M_s$ and we might expect this to be substituted for M_s in the above expression valid for ferromagnets. We shall see that this is indeed true for one type of scattering.

After rather lengthy calculation similar to that of Appendix A of reference 13, the two scattering potentials are found to be

$$\mathcal{H}_1'=[8\pi^2R^3g\beta M_s(u_0+v_0)/3V]\sum_{k\neq0}(u_k+v_k) \times[j_1(kR)/kR](\alpha_0^\dagger\alpha_k+\alpha_k^\dagger\alpha_0), \qquad \text{(III.18)}$$

and

$$\mathcal{H}_2'=[8\pi^2R^3g\beta M_s(u_0+v_0)/V]\sum_{k\neq0}\sin^2\theta_k(u_k+v_k) \times[j_1(kR)/kR](\alpha_0^\dagger\alpha_K+\alpha_k^\dagger\alpha_0), \qquad \text{(III.19)}$$

where $j_1(kR)$ is a spherical Bessel function. $\mathcal{H}_1'$ represents scattering to the lower branch which isotropic in the absence of external fields. The potential $\mathcal{H}_2'$ is for scattering to the anisotropic spin-wave branch. Using the density of states (III.9), the linewidth arising from $\mathcal{H}_1'$ is given by

$$\Delta H=2\pi M_s(a/r_0)(H_e/8\pi M_s)^{1/2} \times\cos^2[(R/a)(8\pi H_AM_s/H_e^2)]. \qquad \text{(III.20)}$$

Then for $R\approx10^{-3}$ cm, as in the experiments by Johnson and Nethercot[14] on MnF_2, $r_0\approx10^{-2}$ cm, and $M_s=600$ Oe, this gives a linewidth of about 0.1 Oe. This is a rather small contribution to the linewidth because the degenerate modes occur at $|k|\approx10^5$ cm^{-1} and are not spread over an appreciable region of k space. Then, $kR\gg1$ and the scattering matrix element is small because we are in the asymptotic tail of the Bessel function. It is important to have degenerate states near $kR\approx1$, where the matrix element is maximum. Such states usually exist for the anisotropic spin-wave branch. Then, using $\mathcal{H}_2'$ with the appropriate density of states, (III.8), we find the result

$$\Delta H=(\pi M_s/8)(R/r_0)(2H_A/H_e)^{1/2}. \qquad \text{(III.21)}$$

For the above parameters, this contributes several Oe to the linewidth.

We have seen that imperfections in antiferromagnets may be about as effective in broadening the AFR as they are in ferromagnets. However, for antiferromagnets with reasonably large anisotropy fields ($H_A\approx10^4-10^5$

[14] F. M. Johnson and A. H. Nethercot, Phys. Rev. 114, 705 (1959).

Oe) as in MnF_2 and FeF_2, the experimental indications[14,15] are that the linewidths are of the order of several hundred to several thousand Oe, much greater than any of the imperfection widths calculated here. Elsewhere,[16] it has been shown that various magnon-phonon processes may be important relaxation mechanisms for such substances where the resonance frequency is in the far infrared. However, several antiferromagnets have recently been found[17,18] (e.g., $KMnF_3$, $RbMnF_3$) with sufficiently low anisotropy fields to bring the resonance to the microwave region. For these materials, the linewidths may be sufficiently small that imperfections can play a dominant role at low temperatures as they do in yttrium iron garnet.

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APPENDIX

In this Appendix, we derive the dispersion law (II.11) using the microscopic form for the dipole-dipole interaction,

$$\mathcal{H}_D=\tfrac{1}{2}g\beta^2\sum_{l,m}[(\mathbf{S}_l\cdot\mathbf{S}_m) -3(\mathbf{S}_l\cdot\mathbf{R}_{lm})(\mathbf{S}_m\cdot\mathbf{R}_{lm})R_{lm}^{-2}]R_{lm}^{-3}, \quad \text{(A1)}$$

where R_{lm} is the vector joining the spins at sites l and m on a cubic lattice. We split this interaction into three parts,

$$\mathcal{H}_D=\mathcal{H}_\uparrow+\mathcal{H}_\downarrow+\mathcal{H}_{\uparrow\downarrow}, \quad \text{(A2)}$$

where $\mathcal{H}_\uparrow$ and $\mathcal{H}_\downarrow$ are the interactions among the spins within a given sublattice and $\mathcal{H}_{\uparrow\downarrow}$ is the interaction between the sublattices. We introduce the creation and destruction operators $a_k^\dagger$, a_k, $b_k^\dagger$, and b_k by (III.12), and substitute them into (A1) to give

$$\mathcal{H}_\uparrow=\tfrac{1}{2}g\beta^2S\sum_{l,k}\left[\left(1-\frac{3}{2}\frac{R_l^+R_l^-}{R_l^2}\right)e^{iR_l\cdot k}(2a_k^\dagger a_k+1) -\tfrac{3}{2}(R_l^-/R_l)^2a_ka_ke^{iR_l\cdot k}+\text{c.c.}\right]R_l^{-3}, \quad \text{(A3)}$$

and

$$\mathcal{H}_{\uparrow\downarrow}=g\beta^2S\sum_{l,k}[(a_kb_k+a_k^\dagger b_k^\dagger)(1-\tfrac{3}{2}R_l^-R_l^+)e^{-\mathbf{k}\cdot\mathbf{R}_l} -\tfrac{3}{2}(a_kb_k+R_l^{-2}e^{-i\mathbf{k}\cdot\mathbf{R}_l}+\text{c.c.})]R_l^{-3}, \quad \text{(A4)}$$

where "c.c." denotes complex conjugate, and in (A3) the sum over l means summation over all sites on one sublattice and in (A4) indicates summation over all sites. If we replace the sums over lattice sites by integrations and integrate by parts, we obtain

$$\begin{aligned}\mathcal{H}_\uparrow&=2\pi g\beta M_s\textstyle\sum_k(k^-k^+/k^2)a_k^\dagger a_k+\tfrac{1}{2}(k^-/k)^2a_ka_{-k} +\tfrac{1}{2}(k^+/k)^2a_k^\dagger a_{-k}^\dagger,\\ \mathcal{H}_\downarrow&=2\pi g\beta M_s\textstyle\sum_k(k^-k^+/k^2)b_k^\dagger b_k+\tfrac{1}{2}(k^-/k)^2b_k^\dagger b_{-k}^\dagger +\tfrac{1}{2}(k^+/k)^2b_kb_{-k}, \quad \text{(A5)}\\ \mathcal{H}_{\uparrow\downarrow}&=2\pi g\beta M_s\textstyle\sum_k\tfrac{1}{2}k^-k^+k^{-2}(a_kb_k+a_k^\dagger b_k^\dagger) +(k^-/k)^2a_kb_{-k}^\dagger+(k^+/k)^2a_k^\dagger b_{-k}.\end{aligned}$$

The complete Hamiltonian including exchange, anisotropy, external field, and dipolar terms may then be written as

$$\begin{aligned}\mathcal{H}=\textstyle\sum_k\{&[A(\mathbf{k})-g\beta H]b_k^\dagger b_k+[A(\mathbf{k})+g\beta H]a_k^\dagger a_k\\ &+B(\mathbf{k})(a_kb_k+a_k^\dagger b_k^\dagger)+C(\mathbf{k})a_ka_k\\ &+C^*(\mathbf{k})a_k^\dagger a_{-k}^\dagger+C(\mathbf{k})b_k^\dagger b_k^\dagger+C^*(\mathbf{k})b_kb_{-k}\\ &+2C(\mathbf{k})a_kb_{-k}^\dagger+2C^*(\mathbf{k})a_k^\dagger b_{-k}\}, \quad \text{(A6)}\end{aligned}$$

where

$$\begin{aligned}A(\mathbf{k})&=2JSz+g\beta H_A+2\pi g\beta M_sk^-k^+k^{-2},\\ B(\mathbf{k})&=2JSz\gamma_k+\pi g\beta M_sk^-k^+k^{-2},\\ C(\mathbf{k})&=\pi g\beta M_s(k^-/k)^2, \quad \text{(A7)}\end{aligned}$$

and

$$\gamma_k=(1/z)\textstyle\sum_\delta e^{i\mathbf{k}\cdot\delta},$$

the last sum being over all nearest neighbors to a given spin. The Hamiltonian (A6) is then easily diagonalized to give the spectrum (II.11).

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